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Letter to the Editors

Notes on the article "Numerical Laplace Transform Inversion of a Function Arising in Viscoelasticity"

In his paper "Numerical Laplace Transform Inversion of a Function Arising in Viscoelasticity" [1] I. M. Longman gives tables of inverse Laplace transform of functions

$$\bar{g}(p) = (1/p)\exp\{-p/(1 + \sigma p)^{1/2}\} \tag{1}$$

for various values of the parameter σ.

The author employs an effective method of calculation of the Padé table. However series expansion of the function $p\bar{g}(p)$ has no normal Padé table for $\sigma = 1$, and the values of the inversion function $g(t)$ for $\sigma = 1$ are not given in the paper. However, in this new paper [2] Longman uses nonlinear sequence to sequence transformation due to D. Levin. Here the case $\sigma = 1$ is computed.

A similar problem has been solved.[1] For the inversion of function (1) we have used the method of [3] according to which the inversion function $g(t)$ may be approximated by

$$g(t) = (\ln 2/t) \sum_{i=1}^{N} V_i \bar{g}(i \ln 2/t). \tag{2}$$

The constants V_i are determined by the expression

$$V_i = (-1)^{i+N/2} \sum_{k=[(i+1)/2]}^{\min(i,N/2)} \frac{k^{N/2}(2k)!}{(N/2 - k)!\,k!\,(k-1)!\,(i-k)!\,(2k-i)!}, \tag{3}$$

where the symbol [ ] designates the integer part of the number. The optimum value of N is approximately equal to the number of digits (word length) used in the computer [3]. In our case $N = 16$ and $N = 18$ were used (word length 64 bits).

For $N = 18$ the values of the function $g(t)$ calculated by using this method coincide with Longman's values [2] for $\sigma \geqslant 0.3$ and $t > 0.3$ up to five significant digits. For $\sigma = 0.2$ the deviations are at the fourth decimal place; for $\sigma = 0.1$ both methods coincide within one or two places. For $N = 16$ the results obtained by both methods are identical for $\sigma \geqslant 0.4$ (also up to five digits).

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For low values of the parameter σ Longman's method gives obviously better results if the usual word length of 32–64 bits is used, since for $\sigma = 0$ the function $g(t)$ converges to Heaviside function $H(t - 1)$ and for the cases of such sudden changes in the inverse functions the expression (2) is not suitable. Longman's method provides for these cases convergence control by comparing the element values on the leading diagonal of the Padé table after inversion.

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